Error Control for Local Broadcasting in Heterogeneous Wireless Ad Hoc Networks

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***Abstract*—Local (single-hop) broadcasting is widely employed in distributed protocols (e.g., neighbor discovery, local information exchange in distributed network optimization and gossip-based algorithms) in wireless ad hoc networks. The performance of local broadcasting is characterized by the mean number of neighbors and the probability distribution of the number of neighbors of a broadcasting node. In this paper, we study the performance of local broadcasting in heterogeneous wireless ad hoc networks in which inter-system interference dominates signal reception quality, considering general fading distributions of both the desired signal and the interfering signals. In addition, we investigate the impacts of different error control techniques (e.g., simple retransmission, Chase combining, and incremental redundancy) on the performance of local broadcasting. The increase in the mean number of neighbors of a broadcasting node with respect to the number of retransmissions of a message is clarified, facilitating QoS provisioning in reliable local broadcasting in interference-limited heterogeneous wireless ad hoc networks.**

***Index Terms*—Local broadcasting, error control, HARQ, interference, heterogeneous networks, ad hoc networks, cognitive radio networks.**

I. INTRODUCTION

LOCAL (single-hop) broadcasting is referred to as the one-hop point-to-multipoint (one-to-many) transmission in wireless ad hoc networks. Local broadcasting occurs in many distributed network protocols, such as neighbor discovery or routing table construction in establishing network connectivity, local information exchange with all one-hop neighbors in distributed network optimization [1] and gossip-based algorithms [2], and the rate of information dissemination [3]. Furthermore, a global broadcast (i.e., a message is required to be disseminated to all nodes in a multi-hop network) consists of a sequence of local broadcasts. The performance of local broadcasting can be characterized by the mean number of neighbors[1] and the probability distribution of the number of neighbors of a broadcasting node.

Existing research is confined to the local broadcasting in a stand-alone wireless ad hoc network without interference [4], [5] and with intra-system interference [6]. However, coexistence of multiple heterogeneous wireless networks emerges in the next generation wireless networking, and several challenges are introduced. One of the major challenges is that inter-system interference from different radio access technologies operating at the same spectrum may significantly degrade the quality of signal reception. Investigations of inter-system interference in coexisting heterogeneous wireless networks have focused on spectrum sharing in two-tier femtocell networks [7], cellular and ad hoc networks [8], narrowband and ultra-wideband networks [9], and cognitive radio networks [10]. More details can be found in [11]–[13].

Following this trend, in this paper we study the performance of local broadcasting in an interference-limited environment consisting of multiple heterogeneous wireless ad hoc networks. We explore the impacts of different error control techniques (including simple retransmission, Chase combining, and incremental redundancy [14], [15]) on the mean number of neighbors and the probability distribution of the number of neighbors in local broadcasting. With the probability distribution of the number of neighbors, QoS provisioning in local broadcasting can be facilitated. An interesting question that can be raised is how many times a source node should broadcast a message without the aid of acknowledgment feedback (e.g. ACK/NACK) so that with a guaranteed probability the message will be successfully received by more than a certain number of nodes. Via the probability distribution, we may answer as follows: "A source node should broadcast a message m times so that with probability η more than j nodes will receive the message successfully".

Finally, cognitive radio networking (CRN) is recognized as a promising technology to enhance spectrum utilization by opportunistically accessing the licensed spectrum [16], [17]. In CRN, secondary (unlicensed) users embedded with cognitive radios coexist with primary (licensed) users. However, secondary users are considered of lower priority in spectrum access and their interference to primary users should be limited. We therefore extend our results to the local broadcasting in CRN with users of different priorities, and the power allocation and rate assignment among secondary users are discussed.

The remainder of this paper is organized as follows. Section II provides background of local broadcasting in a noise-limited environment. In Section III, we present the interference-limited network model and the analytical methods. Analysis of different error control techniques is provided in Section IV. In Section V, the results are applied to local broadcasting in CRN. Numerical results are presented in Section VI. Section VII gives the conclusion.

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[1] A receiving node that can successfully decode the message (at a given information rate) of a transmitting node is a neighbor of the transmitting node.

II. Background: Local broadcasting in a Noise-Limited Environment

We consider a stand-alone wireless ad hoc network, in which the spatial distribution of nodes is assumed to follow a homogeneous Poisson point process (PPP) with density λ. Let $\Phi = \{X_k : k \in \mathbb{N}\}$ denote the set of locations of the nodes and let P denote the transmit power of a node. A pair of nodes are connected if the capacity of the channel between them can support an information rate R, i.e.,

$$\log\left(1+\frac{GPr^{-\alpha}}{N_0}\right) \geq R, \tag{1}$$

where G denotes the fading gain, r is the distance between the pair of nodes, α is the path loss exponent, and N_0 is the background noise power.

Consequently, in local (single-hop) broadcasting, neighbors of a typical broadcasting node (a reference node located at the origin)[2] are resulted from independent thinning of each node $x \in \Phi$ with probability $\mathbb{P}_{G_x}\left(\log\left(1+\frac{G_x P\|x\|^{-\alpha}}{N_0}\right) \geq R\right)$. G_x denotes the fading gain of the desired link from the typical node to the node at x, which is independently drawn according to some distribution f_G, i.e., the distribution function $f_{G_x}(z) = f_G(z),\ \forall x \in \Phi$. $\|x\|$ denotes the distance between the typical node and the node at x. By the mapping theorem [5], [19] indicating that independent thinning of a PPP is still a PPP, the number of neighbors of the typical node is Poisson distributed with mean β

$$\begin{aligned}\beta &= \mathbb{E}_{\Phi,G_x}\left[\sum_{x\in\Phi}\mathbf{1}\left(\log\left(1+\frac{G_x P\|x\|^{-\alpha}}{N_0}\right)\geq R\right)\right]\\ &= \mathbb{E}_{\Phi}\left[\sum_{x\in\Phi}\mathbb{P}_{G_x}\left(\log\left(1+\frac{G_x P\|x\|^{-\alpha}}{N_0}\right)\geq R\right)\right]\\ &\overset{(a)}{=} \int_0^\infty \lambda \mathbb{P}_G\left(G \geq \frac{(2^R-1)N_0}{Pr^{-\alpha}}\right) 2\pi r dr\\ &= \frac{\lambda\pi P^\delta \mathbb{E}[G^\delta]}{(2^R-1)^\delta N_0^\delta}.\end{aligned} \tag{2}$$

where $\mathbf{1}(\cdot)$ is the indicator function, $\delta = \frac{2}{\alpha}$, and (a) follows Campbell's theorem [18]. The probability that a node has k neighbors is denoted as p_k, which satisfies

$$p_k = \frac{\exp(-\beta)\beta^k}{k!},\ k = 0, 1, 2, \cdots. \tag{3}$$

The probability that a node has no neighbors is $\exp(-\beta)$. Note that in the case without fading, the mean number of neighbors of a broadcasting node is $\frac{\lambda\pi P^\delta}{(2^R-1)^\delta N_0^\delta}$, implying that $\mathbb{E}[G^\delta]$ corresponds to the *connectivity fading gain* [4], [5]. In summary, the mean number of neighbors and the probability distribution of the number of neighbors are respectively β and p_k in local broadcasting in a noise-limited wireless ad hoc network.

[2]By the stationarity of homogeneous PPP [18], the statistics measured by the typical node is representative for all other nodes. Therefore, we focus our discussions on the typical node. A typical node may be a transmitter or a receiver depending on the context.

III. Interference-limited Network Model and Analytical Methods

We consider a wireless network (denoted as WN_0), in which the spatial distribution of nodes is assumed to follow a PPP with density λ. Let $\Phi = \{X_k : k \in \mathbb{N}\}$ denote the set of locations of the nodes in WN_0 and let P denote the transmit power of a node in WN_0. The wireless network WN_0 coexists with N heterogeneous wireless networks (denoted as $WN_i,\ 1 \leq i \leq N$), where the spatial distribution of *active* nodes in WN_i is assumed to follow a PPP with density μ_i. The set of locations of the active nodes in WN_i is denoted as $\Psi_i = \{Y_k^i : k \in \mathbb{N}\}$ and the transmit power of a node is P_i. The main notations used in this paper are summarized in Table I.

The interference from the active nodes in WN_i to a node at $x \in \Phi$ is denoted as $I_{i,x} = \sum_{y\in\Psi_i} G_{yx,i}P_i\|y-x\|^{-\alpha}$. $G_{yx,i}$ denotes the fading gain of the interfering link from the node at $y \in \Psi_i$ to the node at $x \in \Phi$, which is assumed to be independently drawn from f_{G_i}, i.e., the distribution function $f_{G_{yx,i}}(z) = f_{G_i}(z),\ \forall y \in \Psi_i, x \in \Phi$. $\|y-x\|$ is the distance between the node at y and the node at x, and α is the path loss exponent. By the stationarity of homogeneous PPP [18], the statistics of interference measured by a typical node (i.e., a reference node located at the origin) is representative for all other nodes. We define $I_i = \sum_{y\in\Psi_i} G_{y0,i}P_i\|y\|^{-\alpha}$, where $G_{y0,i}$ denotes the fading gain between the node at $y \in \Psi_i$ and the typical node of WN_0 at origin. The statistics of I_i and $I_{i,x}$ are the same. All packet transmissions of the networks are assumed to be slotted and synchronized, and each link's fading gain is assumed to pick up a different realization in each slot (i.e., the channel coherence time is about one slot).

In local (single-hop) broadcasting, a node at $x \in \Phi$ successfully receives the message from a typical broadcasting node (i.e., the node at x is a neighbor of the typical broadcasting node) if the capacity of the channel between them can support an information rate R, i.e.,

$$\log\left(1+\frac{G_x P\|x\|^{-\alpha}}{\sum_{i=1}^N I_{i,x}}\right) \geq R, \tag{4}$$

where G_x denotes the fading gain of the desired link from the typical node to the node at $x \in \Phi$, which is assumed to be independently drawn from f_G, i.e., the distribution function $f_{G_x}(z) = f_G(z),\ \forall x \in \Phi$. $\|x\|$ denotes the distance between the typical node and the node at x. Since WN_0 operates in an interference-limited environment, we ignore the background noise. An illustration is provided in Fig. 1.

The mean number of neighbors of the typical broadcasting node is denoted as $\widetilde{\beta}$. We have (5), where (a) follows that expectation is a linear operator that commutes with summation. Therefore, the correlation among interferences observed at different nodes $x \in \Phi$ does not matter. (b) follows the stationarity of homogeneous PPP. (c) follows Campbell's theorem [18]. (d) follows from [20], where $g(t)$ is the inverse Laplace transform of the complementary cumulative distribution function (ccdf) of the fading gain of the desired link $\overline{F}_G(s) = \mathbb{P}(G \geq s)$. $\Gamma(z) = \int_0^\infty t^{z-1}e^{-t}dt$ is the Gamma function.

TABLE I
MAIN NOTATIONS

Description	Notation	
	Ad hoc network WN_0	Interfering networks $WN_i,\ 1 \leq i \leq N$
Node density	λ	μ_i
The set of locations of nodes	Φ	Ψ_i
Transmit power	P	P_i
Fading gain	$G_x : x \in \Phi$, $G_x \sim f_G$	$G_{yx,i} : y \in \Psi_i, x \in \Phi$, $G_{yx,i} \sim f_{G_i}$
Inverse Laplace transform of ccdf of G	$g(t)$	
Information rate	R	
Mean number of neighbors (noise-limited)	β	
Mean number of neighbors (interference-limited)	$\widetilde{\beta}$	
Path loss exponent	$\alpha,\ \delta = 2/\alpha$	
Background noise power	N_0	

$$\begin{aligned}
\widetilde{\beta} &= \mathbb{E}_{\Phi,G_x,\Psi_i,G_{yx,i}}\left[\sum_{x\in\Phi}\mathbf{1}\left(\log\left(1+\frac{G_xP\|x\|^{-\alpha}}{\sum_{i=1}^N\sum_{y\in\Psi_i}G_{yx,i}P_i\|y-x\|^{-\alpha}}\right)\geq R\right)\right]\\
&\overset{(a)}{=} \mathbb{E}_{\Phi}\left[\sum_{x\in\Phi}\mathbb{P}_{G_x,\Psi_i,G_{yx,i}}\left(\log\left(1+\frac{G_xP\|x\|^{-\alpha}}{\sum_{i=1}^N\sum_{y\in\Psi_i}G_{yx,i}P_i\|y-x\|^{-\alpha}}\right)\geq R\right)\right]\\
&\overset{(b)}{=} \mathbb{E}_{\Phi}\left[\sum_{x\in\Phi}\mathbb{P}_{G,\Psi_i,G_{y0,i}}\left(\log\left(1+\frac{GP\|x\|^{-\alpha}}{\sum_{i=1}^N\sum_{y\in\Psi_i}G_{y0,i}P_i\|y\|^{-\alpha}}\right)\geq R\right)\right]\\
&\overset{(c)}{=} \int_0^\infty \lambda\mathbb{P}_{G,I_i}\left(G\geq\frac{(2^R-1)\sum_{i=1}^N I_i}{Pr^{-\alpha}}\right)2\pi r dr\\
&\overset{(d)}{=} \frac{\lambda P^\delta}{\left(\sum_{i=1}^N\mu_iP_i^\delta\mathbb{E}[G_i^\delta]\right)(2^R-1)^\delta\Gamma(1-\delta)}\int_0^\infty\frac{g(t)}{t^\delta}dt
\end{aligned}\tag{5}$$

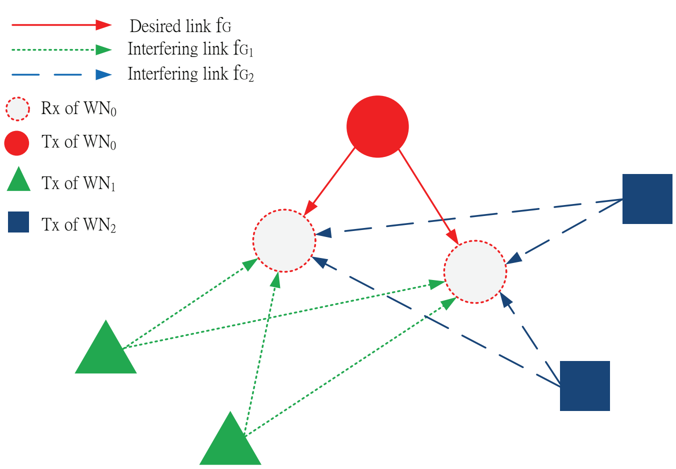

Fig. 1. A transmitter (Tx) in WN_0 broadcasts a message while receivers (Rx) in WN_0 sustain interference from heterogeneous networks WN_1 and WN_2. The fading gain of the desired link is drawn according to distribution f_G, and the fading gain of the interfering link from WN_1 (WN_2) is drawn according to distribution f_{G_1} (f_{G_2}).

Lemma 1 ([20]). *$\widetilde{\beta}$ can be bounded from above by*

$$\widetilde{\beta} < \frac{\lambda P^\delta\mathbb{E}[G^\delta](2^{1+\delta}-1)}{\left(\sum_{i=1}^N\mu_iP_i^\delta\mathbb{E}[G_i^\delta]\right)(2^R-1)^\delta 2^{2\delta}} \triangleq \widetilde{\beta}^{Upper}. \tag{6}$$

Proof: The proof follows the lines in [20]. ■

Lemma 2 ([20]). *$\widetilde{\beta}$ can be bounded from below by*

$$\widetilde{\beta} \geq \frac{\lambda P^\delta\mathbb{E}[G^\delta](1-\frac{2}{\alpha-2})^+}{\left(\sum_{i=1}^N\mu_iP_i^\delta\mathbb{E}[G_i^\delta]\right)(2^R-1)^\delta} \triangleq \widetilde{\beta}^{Lower}, \tag{7}$$

where $(x)^+ \triangleq \max(x,0)$. The lower bound is non-trivial when $\alpha > 4$.

Proof: The proof is presented in Appendix A. ■

IV. Error Control Techniques

In the following subsections, we discuss the impacts of important error control techniques on the performance metric $\widetilde{\beta}$ to facilitate QoS provisioning in local broadcasting. We assume that all fading gains of the desired links and the interfering links are exponentially distributed (Rayleigh fading) with unit average power and are i.i.d. across retransmission slots. In addition, the locations of all nodes (receivers in Φ and interferers in Ψ_i) are supposed to be static across retransmission slots.

$$\widetilde{\beta}^{\text{Re}}(m) = \mathbb{E}_{\Phi, G_x^{(k)}, \Psi_i, G_{yx,i}^{(k)}} \left[\sum_{x\in\Phi} \left\{ 1 - \prod_{k=1}^{m} \left[1 - \mathbf{1}\left(\log\left(1 + \frac{G_x^{(k)} P \|x\|^{-\alpha}}{\sum_{i=1}^{N} \sum_{y\in\Psi_i} G_{yx,i}^{(k)} P_i \|y-x\|^{-\alpha}} \right) \geq R \right) \right] \right\} \right] \tag{8}$$

A. Simple retransmission

Under simple retransmission scheme, a retransmitted message that cannot be successfully decoded at the receiver side is dropped. The average number of neighbors of a typical broadcasting node, who receive the message successfully at least one time in total m retransmissions, is denoted as $\widetilde{\beta}^{\text{Re}}(m)$ and we have (8).

In order to further facilitate the analysis, we modify the assumption on the mobility of interferers. Here in the following proposition, the interferers are assumed to have infinite mobility, and we use $\widehat{\beta}^{\text{Re}}(m)$ to denote such approximated quantity.

Proposition 1. *Under simple retransmission scheme with m retransmissions and with interferers having infinite mobility, $\widehat{\beta}^{Re}(m)$ is obtained as*

$$\widehat{\beta}^{Re}(m) = \frac{\lambda P^{\delta}}{\left(\sum_{i=1}^{N} \mu_i P_i^{\delta} \Gamma(1+\delta)\right)(2^R-1)^{\delta}\Gamma(1-\delta)} \sum_{k=1}^{m} \frac{1}{k}. \tag{9}$$

Proof: The proof is presented in Appendix B. ■

This proposition indicates the increase in the mean number of neighbors $\widehat{\beta}^{\text{Re}}(m)$ with respect to the number of retransmissions m under simple retransmission scheme. The harmonic series $\sum_{k=1}^{m} \frac{1}{k}$ has a logarithmic growth, i.e., when m approaches infinity, $\sum_{k=1}^{m} \frac{1}{k} = \ln\, m + \gamma$, where γ is the Euler's constant.

B. Chase combining

Instead of dropping the retransmitted message that cannot be successfully decoded, packet combining technique (Chase combining) widely used in hybrid ARQ [14] can be adopted at the receiving node. Chase combining is equivalent to performing maximal ratio combining (MRC) at symbol level in different retransmission slots. At retransmission slot k, the received signal at $x \in \Phi$ (denoted as $s_x^{(k)}$) is

$$\begin{aligned} s_x^{(k)} &= h_x^{(k)} \sqrt{P} \|x\|^{-\alpha/2} b^{(k)} \\ &\quad + \sum_{i=1}^{N} \sum_{y\in\Psi_i} h_{yx,i}^{(k)} \sqrt{P_i} \|y-x\|^{-\alpha/2} b_{y,i}^{(k)} \\ &\quad + n_x^{(k)},\ k = 1, \cdots, m, \end{aligned} \tag{10}$$

where $\|x\|$ is the distance between the typical broadcasting node and the node at x, $n_x^{(k)}$ is zero mean complex AWGN with variance N_0, $h_x^{(k)}$ and $h_{yx,i}^{(k)}$ are respectively the channel amplitude coefficient of the desired link and the interfering link at retransmission slot k, $b^{(k)}$ and $b_{y,i}^{(k)}$ are respectively the desired symbol and the symbol from the interferer at $y \in \Phi_i$ at retransmission slot k. Note that $b^{(1)} = \cdots = b^{(m)} \triangleq b$ since the same desired symbol is retransmitted at each slot. We also assume that $|b| = |b_{y,i}^{(k)}| = 1$.

[3]Under Chase combining, the received signal $s_x^{(k)}$ is then weighted by the channel coefficient $h_x^{(k)*}$

$$\begin{aligned} h_x^{(k)*} s_x^{(k)} &= |h_x^{(k)}|^2 \sqrt{P} \|x\|^{-\alpha/2} b \\ &\quad + \sum_{i=1}^{N} \sum_{y\in\Psi_i} h_x^{(k)*} h_{yx,i}^{(k)} \sqrt{P_i} \|y-x\|^{-\alpha/2} b_{y,i}^{(k)} \\ &\quad + h_x^{(k)*} n_x^{(k)},\ k = 1, \cdots, m. \end{aligned} \tag{11}$$

Let $\mathbf{h}_x = [h_x^{(1)} \cdots h_x^{(m)}]^T$ and $\mathbf{h}_{yx,i} = [h_{yx,i}^{(1)} \cdots h_{yx,i}^{(m)}]^T$, the received SINR after Chase combining can be expressed as

$$\begin{aligned} &\frac{\|\mathbf{h}_x\|^4 P \|x\|^{-\alpha}}{\sum_{i=1}^{N} \sum_{y\in\Psi_i} |\mathbf{h}_x^H \mathbf{h}_{yx,i}|^2 P_i \|y-x\|^{-\alpha} + \|\mathbf{h}_x\|^2 N_0} \\ &= \frac{\|\mathbf{h}_x\|^2 P \|x\|^{-\alpha}}{\sum_{i=1}^{N} \sum_{y\in\Psi_i} \left| \frac{\mathbf{h}_x^H \mathbf{h}_{yx,i}}{\|\mathbf{h}_x\|} \right|^2 P_i \|y-x\|^{-\alpha} + N_0}. \end{aligned} \tag{12}$$

Since we assume that all links are Rayleigh faded with unit average power, the elements of $\mathbf{h}_x$ and $\mathbf{h}_{yx,i}$ are i.i.d. circularly symmetric complex Gaussian random variables with zero mean and unit variance (denoted as $\mathcal{CN}(0,1)$). By using the fact that a linear combination of Gaussian random variables is still Gaussian, it follows [21] that $\frac{\mathbf{h}_x^H \mathbf{h}_{yx,i}}{\|\mathbf{h}_x\|} \sim \mathcal{CN}(0,1)$, and $\frac{\mathbf{h}_x^H \mathbf{h}_{yx,i}}{\|\mathbf{h}_x\|}$ and $\mathbf{h}_x$ are independent. In addition, $\left|\frac{\mathbf{h}_x^H \mathbf{h}_{yx,i}}{\|\mathbf{h}_x\|}\right|^2$ is exponentially distributed with unit mean, and $\|\mathbf{h}_x\|^2 = \sum_{k=1}^{m} |h_x^{(k)}|^2$ is a sum of exponential random variables with unit mean.

Successful decoding at $x \in \Phi$ after m retransmissions with Chase combining occurs when

$$\log\left(1 + \frac{\|\mathbf{h}_x\|^2 P \|x\|^{-\alpha}}{\sum_{i=1}^{N} I_{i,x}} \right) \geq R, \tag{13}$$

where $I_{i,x} = \sum_{y\in\Psi_i} \left| \frac{\mathbf{h}_x^H \mathbf{h}_{yx,i}}{\|\mathbf{h}_x\|} \right|^2 P_i \|y-x\|^{-\alpha}$, and the background noise power N_0 is ignored. The mean number of neighbors of a typical broadcasting node of WN_0 is denoted as $\widetilde{\beta}^{\text{CC}}(m)$ and we have (14).

Proposition 2. *Under Chase combining scheme with m retransmissions, $\widetilde{\beta}^{CC}(m)$ is obtained as*

$$\begin{aligned} \widetilde{\beta}^{CC}(m) &= \frac{\lambda P^{\delta}}{\left(\sum_{i=1}^{N} \mu_i P_i^{\delta} \Gamma(1+\delta)\right)(2^R-1)^{\delta}\Gamma(1-\delta)} \\ &\quad \cdot \sum_{k=0}^{m-1} \frac{\Gamma(\delta+k)}{\Gamma(\delta)\Gamma(k+1)}. \end{aligned} \tag{15}$$

Proof: The proof is presented in Appendix C. ■

This proposition indicates the increase in the mean number of neighbors $\widetilde{\beta}^{\text{CC}}(m)$ with respect to the number of retransmissions m under Chase combining scheme. It can be shown that $\widetilde{\beta}^{\text{CC}}(m)$ is asymptotically equivalent to m^{δ}, as m increases.

[3]In the following, $(\cdot)^T$ stands for transpose, $(\cdot)^*$ stands for conjugate, and $(\cdot)^H$ stands for transpose conjugate.

$$\widetilde{\beta}^{\mathrm{CC}}(m) = \mathbb{E}_{\Phi, h_x^{(k)}, \Psi_i, h_{yx,i}^{(k)}} \left[\sum_{x\in\Phi} \mathbf{1} \left(\log \left(1 + \frac{\|\mathbf{h}_x\|^2 P \|x\|^{-\alpha}}{\sum_{i=1}^N \sum_{y\in\Psi_i} \left| \frac{\mathbf{h}_x^H \mathbf{h}_{yx,i}}{\|\mathbf{h}_x\|} \right|^2 P_i \|y-x\|^{-\alpha}} \right) \geq R \right) \right] \tag{14}$$

C. Incremental redundancy

Rather than retransmitting the same message, the source node can transmit incremental redundancy [14] of the source message at each retransmission slot. Code combining (joint decoding of the source message and all received incremental redundancy of the source message) is performed at the receiving node. Successfully decoding after m retransmissions with code combining occurs when

$$\sum_{k=1}^{m} \log \left(1 + \frac{G_x^{(k)} P \|x\|^{-\alpha}}{\sum_{i=1}^N I_{i,x}} \right) \geq R, \tag{16}$$

where $I_{i,x} = \sum_{y\in\Psi_i} G_{yx,i}^{(k)} P_i \|y-x\|^{-\alpha}$. The mean number of neighbors of a typical broadcasting node of WN_0 is denoted as $\widetilde{\beta}^{\mathrm{IR}}(m)$. We have (17), where we use Jensen's inequality in the derivation

$$\sum_{k=1}^{m} \log(1+x_k) \leq m \log \left(1 + \frac{1}{m} \sum_{k=1}^{m} x_k \right), \tag{18}$$

where $x_k,\ 1 \leq k \leq m$, are nonnegative. Please note that the upper bound is tight in the low SIR regime, in which error control is indeed necessary. The reason is that the upper bound is based on Jensen's inequality $\sum_{k=1}^{m} \log(1+x_k) \leq m \log\left(1 + \frac{1}{m}\sum_{k=1}^{m} x_k\right)$, where the logarithm function can be approximated by a linear function (i.e., $\log(1+x) \approx x \log e$) when $x \approx 0$ (i.e., low SIR regime), and equality (in Jensen's inequality) holds for linear functions.

In order to further facilitate the analysis, we modify the assumption on the fading gains of the interfering links. In the following proposition, the fading gains of the interfering links are assumed to be fixed during retransmissions, and we use $\widehat{\beta}^{\text{IR, Upper}}(m)$ to denote such approximated quantity.

Proposition 3. *Under incremental redundancy scheme with m retransmissions and fixed fading gains of the interfering signals, $\widehat{\beta}^{\text{IR, Upper}}(m)$ is obtained as*

$$\begin{aligned} &\widehat{\beta}^{\text{IR,Upper}}(m) \\ &= \frac{\lambda P^\delta}{(\sum_{i=1}^N \mu_i P_i^\delta \Gamma(1+\delta)) m^\delta (2^{R/m}-1)^\delta \Gamma(1-\delta)} \\ &\cdot \sum_{k=0}^{m-1} \frac{\Gamma(\delta+k)}{\Gamma(\delta)\Gamma(k+1)}. \end{aligned} \tag{19}$$

Proof: The proof is presented in Appendix D. ∎

This proposition indicates the increase in the mean number of neighbors $\widehat{\beta}^{\text{IR, Upper}}(m)$ with respect to the number of retransmissions m under incremental redundancy scheme. It can be shown that $\widehat{\beta}^{\text{IR, Upper}}(m)$ is asymptotically equivalent to m^δ, as m increases.

D. Probability distribution of the number of neighbors

To obtain the probability distribution of the number of neighbors of a typical broadcasting node, we ignore the spatial correlation of interference[4] observed at different nodes in Φ. As a result, neighbors of the typical broadcasting node are resulted from *independent* thinning of each node $x \in \Phi$ with probability $\mathbb{P}_{G_x, I_{i,x}}\left(\log\left(1 + \frac{G_x P \|x\|^{-\alpha}}{\sum_{i=1}^N I_{i,x}}\right) \geq R\right)$. By the mapping theorem [5], the number of neighbors is Poisson distributed with mean $\widetilde{\beta} = \int_0^\infty \lambda \mathbb{P}_{G_x, I_{i,x}}\left(\log\left(1 + \frac{G_x P r^{-\alpha}}{\sum_{i=1}^N I_{i,x}}\right) \geq R\right) 2\pi r dr$, i.e., the probability that a node has k neighbors is $\widetilde{p}_k = \frac{\exp(-\widetilde{\beta})\widetilde{\beta}^k}{k!},\ k = 0, 1, 2, \cdots$. Similar conclusions hold for local broadcasting with different error control techniques, facilitating QoS provisioning. For example, under simple retransmission scheme, if we want to ensure with probability η that more than j nodes can successfully receive a message without the aid of acknowledgment feedback (e.g. ACK/NACK), the number of retransmissions m is the smallest integer satisfying $\sum_{k=j}^\infty \exp(-\widetilde{\beta}^{\mathrm{Re}}(m)) \widetilde{\beta}^{\mathrm{Re}}(m)^k / k! \geq \eta$.

V. An Application to Cognitive Radio Network

In cognitive radio network (CRN), secondary (unlicensed) users (SUs) embedded with cognitive radios coexist with primary (licensed) users (PUs). On the one hand, the received signal at a secondary receiver (SR) is interfered with by primary transmitters (PTs); on the other hand, the signal transmission of a secondary transmitter (ST) should not violate the interference temperature constraints at primary receivers (PRs) [16], [22], indicating the maximum tolerable interference power at a PR. We subsequently discuss the impacts of these two restrictions on the performance of local broadcasting in the CRN.

We assume that the spatial distribution of SUs (resp. PTs) follows a PPP with density λ_{SU} (resp. λ_{PT}). In addition, the transmit power of an SU (resp. a PT) is denoted as P_{SU} (resp. P_{PT}) and the information rate of an SU is denoted as R_{SU}. The fading gain of the desired link from an ST to an SR is denoted as $G_{ST,SR}$, and that of the interfering link from a PT to an SR is denoted as $G_{PT,SR}$. The mean number of neighbors of an SU in local broadcasting is denoted as $\widetilde{\beta}_{SU}$, which can be computed by using **Lemma 1** as

$$\widetilde{\beta}_{SU} < \frac{\lambda_{SU} P_{SU}^\delta \mathbb{E}[G_{ST,SR}^\delta](2^{1+\delta}-1)}{\lambda_{PT} P_{PT}^\delta \mathbb{E}[G_{PT,SR}^\delta](2^{R_{SU}}-1)^\delta 2^{2\delta}}. \tag{20}$$

Exact result and lower bound can be easily obtained by using (5) and **Lemma 2**, respectively.

Let τ_{th} denote the interference temperature (i.e., the maximum permissible interference power) at a PR and let λ_{PR} denote the density of PRs. The probability that the interference power from an ST to a PR exceeds the interference temperature is $\mathbb{P}(G_{ST,PR} P_{SU} r^{-\alpha} > \tau_{th})$, where r denotes

[4] Please note that we do not ignore the spatial correlation of interference in the deviations of (5), **Lemma 1**, **Lemma 2**, and **Proposition 1** to **Proposition 3**. Actually, such correlation is not revealed in these mean statistics.

$$\widetilde{\beta}^{\text{IR}}(m) = \mathbb{E}_{\Phi,G_x^{(k)},\Psi_i,G_{yx,i}^{(k)}}\left[\sum_{x\in\Phi}\mathbf{1}\left(\sum_{k=1}^{m}\log\left(1+\frac{G_x^{(k)}P\|x\|^{-\alpha}}{\sum_{i=1}^{N}\sum_{y\in\Psi_i}G_{yx,i}^{(k)}P_i\|y-x\|^{-\alpha}}\right)\geq R\right)\right]$$
$$\leq \mathbb{E}_{\Phi,G_x^{(k)},\Psi_i,G_{yx,i}^{(k)}}\left[\sum_{x\in\Phi}\mathbf{1}\left(m\log\left(1+\frac{1}{m}\sum_{k=1}^{m}\frac{G_x^{(k)}P\|x\|^{-\alpha}}{\sum_{i=1}^{N}\sum_{y\in\Psi_i}G_{yx,i}^{(k)}P_i\|y-x\|^{-\alpha}}\right)\geq R\right)\right]$$
$$\triangleq \widetilde{\beta}^{\text{IR, Upper}}(m) \quad (17)$$

the distance between the ST and the PR, and $G_{ST,PR}$ denotes the fading gain of the interfering link from the ST to the PR. By the mapping theorem [5], the number of PRs, whose received interference power from an ST exceeds the interference temperature, is Poisson distributed with mean

$$\int_0^{\infty}\lambda_{PR}\mathbb{P}\left(G_{ST,PR}P_{SU}r^{-\alpha}>\tau_{th}\right)2\pi r dr$$
$$=\lambda_{PR}\pi P_{SU}^{\delta}\tau_{th}^{-\delta}\mathbb{E}[G_{ST,PR}^{\delta}]. \quad (21)$$

An SU can be active for broadcasting only when its interference power to each PR is below the interference temperature. Thus, the active probability of an SU (denoted as p_{SU}^a) is

$$p_{SU}^a=\exp(-\lambda_{PR}\pi P_{SU}^{\delta}\tau_{th}^{-\delta}\mathbb{E}[G_{ST,PR}^{\delta}]). \quad (22)$$

A. Optimal power allocation

Observe that when an SU's transmit power P_{SU} decreases, its active probability p_{SU}^a increases; however, its mean number of neighbors $\widetilde{\beta}_{SU}$ decreases. We define the effective mean number of neighbors of an SU (denoted as $\widetilde{\beta}_{eff}$) in local broadcasting as the product of p_{SU}^a and $\widetilde{\beta}_{SU}$, i.e.,

$$\widetilde{\beta}_{eff}=p_{SU}^a\widetilde{\beta}_{SU}$$
$$=\exp(-\lambda_{PR}\pi P_{SU}^{\delta}\tau_{th}^{-\delta}\mathbb{E}[G_{ST,PR}^{\delta}])$$
$$\times\frac{\lambda_{SU}P_{SU}^{\delta}\mathbb{E}[G_{ST,SR}^{\delta}](2^{1+\delta}-1)}{\lambda_{PT}P_{PT}^{\delta}\mathbb{E}[G_{PT,SR}^{\delta}](2^{R_{SU}}-1)^{\delta}2^{2\delta}}. \quad (23)$$

After some calculations, the optimal transmit power is $P_{SU}^*=(\lambda_{PR}\pi\tau_{th}^{-\delta}\mathbb{E}[G_{ST,PR}^{\delta}])^{-\delta^{-1}}$, which maximizes the effective mean number of neighbors of an SU in local broadcasting.

B. Optimal information rate assignment

Observe that when an SU's information rate R_{SU} decreases, its effective mean number of neighbors $\widetilde{\beta}_{eff}$ increases. There exists a trade-off between them. We define the total delivered data rate of an SU in local broadcasting as the product of R_{SU} and $\widetilde{\beta}_{eff}$. That is,

$$R_{SU}\exp(-\lambda_{PR}\pi P_{SU}^{\delta}\tau_{th}^{-\delta}\mathbb{E}[G_{ST,PR}^{\delta}])$$
$$\times\frac{\lambda_{SU}P_{SU}^{\delta}\mathbb{E}[G_{ST,SR}^{\delta}](2^{1+\delta}-1)}{\lambda_{PT}P_{PT}^{\delta}\mathbb{E}[G_{PT,SR}^{\delta}](2^{R_{SU}}-1)^{\delta}2^{2\delta}}. \quad (24)$$

The optimal information rate R_{SU}^* satisfies $2^{R_{SU}^*}-1=R_{SU}^*\delta 2^{R_{SU}^*}\ln 2$, which maximizes the total delivered data rate of an SU in local broadcasting.

VI. Numerical Results

Fig. 2 compares the probability distribution of the number of neighbors of a broadcasting node in a stand-alone wireless network (noise-limited) to that in heterogeneous wireless networks (interference-limited). The system parameters are set as $\lambda=10^{-3}$, $R=4$, $P=0.1$, $N_0=10^{-9}$, $\alpha=4$, $N=1$, $\mu_1=0.5\times10^{-4}$, and $P_1=0.2$. We consider Nakagami-m ($m=3$) fading channel and Rayleigh fading channel (corresponding to $m=1$) with unit average power. Under Nakagami-m fading channels, the mean number of neighbors of a broadcasting node in the stand-alone network is $\beta\approx7.8$, and that in heterogeneous networks is $\widetilde{\beta}\approx2.3$. Under Rayleigh fading channels, we have $\beta\approx7.2$ and $\widetilde{\beta}\approx2.3$. In a noise-limited environment, from (2), we observe that $\beta\propto\mathbb{E}[G^{\delta}]=\frac{\Gamma(\delta+m)}{m^{\delta}\Gamma(m)}$, which increases with m. Therefore, β with Nakagami-m fading channels is greater than β with Rayleigh fading channels.

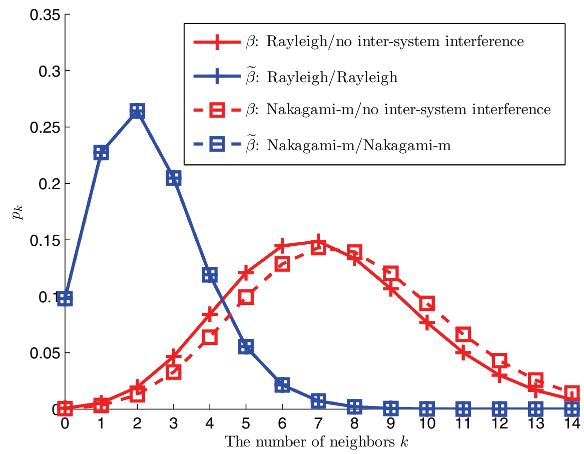

Fig. 2. Plots of the probability mass function p_k v.s. the number of neighbors k. The legend "Rayleigh/no inter-system interference" represents that the desired link is Rayleigh-faded and there is no interfering link. Other cases are similarly defined. The system parameters are set as $\lambda=10^{-3}$, $R=4$, $P=0.1$, $N_0=10^{-9}$, $\alpha=4$, $N=1$, $\mu_1=0.5\times10^{-4}$, and $P_1=0.2$. Both Rayleigh and Nakagami-m ($m=3$) fading are with unit average power.

In Fig. 3, we compare the upper bound $\widetilde{\beta}^{\text{Upper}}$ to the exact value $\widetilde{\beta}$ under different fading distributions. From **Lemma 1**, we observe that $\widetilde{\beta}^{\text{Upper}}$ is proportional to the ratio of $\mathbb{E}[G^{\delta}]$ to $\mathbb{E}[G_i^{\delta}]$, i.e., $\widetilde{\beta}^{\text{Upper}}\propto\mathbb{E}[G^{\delta}]/\mathbb{E}[G_i^{\delta}]$, where G and G_i are the fading gains of the desired signal and interfering signal, respectively. In addition, under Nakagami-m fading channels with unit average power, we have $\mathbb{E}[G^{\delta}]$ (resp. $\mathbb{E}[G_i^{\delta}]$) $=\frac{\Gamma(\delta+m)}{m^{\delta}\Gamma(m)}$, which increases with m for $0<\delta<1$ and $m\geq1$. These two facts together suggest that $\widetilde{\beta}^{\text{Upper}}$ is largest when the desired link is Nakagami-m ($m=3$) faded and the interfering link is Rayleigh ($m=1$) faded. Similar conclusions hold for $\widetilde{\beta}$.

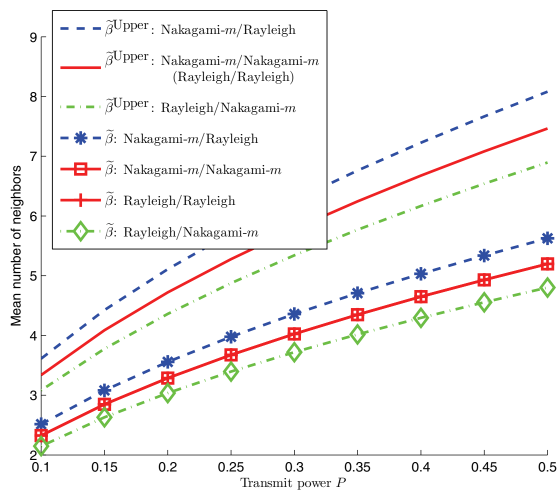

Fig. 3. $\widetilde{\beta}^{\rm Upper}$ and $\widetilde{\beta}$ are compared under different fading distributions of desired link and interfering link as transmit power P increases. The legend, for example, "Nakagami-m/Rayleigh" represents that the desired link is Nakagami-m faded and the interfering link is Rayleigh faded. The system parameters are set as $\lambda = 10^{-3}$, $R = 4$, $\alpha = 4$, $N = 1$, $\mu_1 = 0.5 \times 10^{-4}$, and $P_1 = 0.2$. Both Rayleigh and Nakagami-m ($m = 3$) fading are with unit average power.

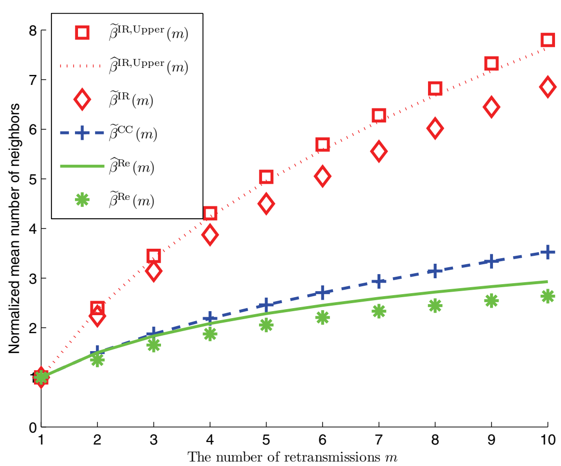

Fig. 4. Plots of the normalized mean number of neighbors v.s. the number of retransmissions m under the simple retransmission scheme $\widetilde{\beta}^{\rm Re}(m)$, the Chase combining scheme $\widetilde{\beta}^{\rm CC}(m)$, and the incremental redundancy scheme $\widetilde{\beta}^{\rm IR}(m)$. The normalization is with respect to $\widetilde{\beta}^{\rm Re}(1)$. The system parameters are set as $\lambda = 10^{-3}$, $R = 4$, $P = 0.1$, $\alpha = 4$, $N = 1$, $\mu_1 = 0.5 \times 10^{-4}$, and $P_1 = 0.2$. All links are Rayleigh faded with unit average power.

Fig. 4 shows the increase in the normalized mean number of neighbors with respect to the number of retransmissions m under different error control techniques. For $\widetilde{\beta}^{\rm Re}(m)$, $\widetilde{\beta}^{\rm CC}(m)$, $\widetilde{\beta}^{\rm IR}(m)$, and $\widetilde{\beta}^{\rm IR,Upper}(m)$, all fading gains of the desired links and the interfering links are i.i.d. across retransmission slots and the locations of all nodes (receivers in Φ and interferers in Ψ_i) are static. In simple retransmission scheme, the approximated quantity $\widehat{\beta}^{\rm Re}(m)$ is based on considering interferers with infinite mobility. It turns out that the approximated quantity overestimates a little, suggesting that mobility of interferers is beneficial to signal reception in Φ because outage is no longer dominated by a nearby interferer. In incremental redundancy scheme, the approximated quantity

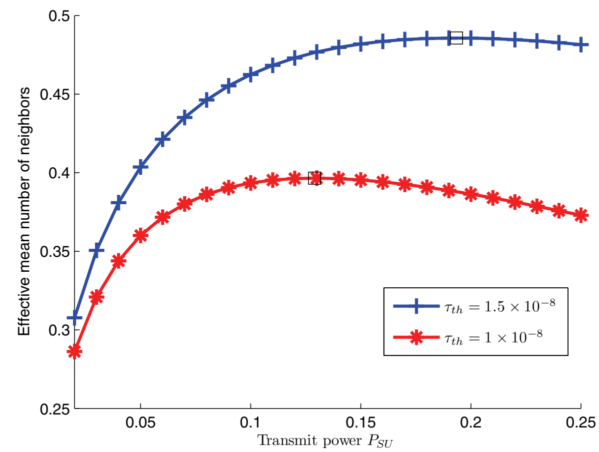

Fig. 5. Plots of the effective mean number of neighbors $\widetilde{\beta}_{eff}$ v.s. transmit power of an SU P_{SU}. The system parameters are set as $\lambda_{SU} = 10^{-3}$, $R_{SU} = 4$, $\alpha = 4$, $\lambda_{PT} = \lambda_{PR} = 10^{-4}$, $P_{PT} = 0.3$, and $\tau_{th} = 1.5 \times 10^{-8}$ or 1×10^{-8}. All links are Rayleigh faded with unit average power.

$\widehat{\beta}^{\rm IR,Upper}(m)$ is based on the assumption that the fading gains of the interfering links are fixed during retransmissions, leading to a small discrepancy. The gap between $\widetilde{\beta}^{\rm IR,Upper}(m)$ and $\widetilde{\beta}^{\rm IR}(m)$ is due to Jensen's inequality.

Fig. 4 also indicates the logarithmic growth of $\widehat{\beta}^{\rm Re}(m)$ as shown in **Proposition 1**, and the limiting behaviors of $\widetilde{\beta}^{\rm CC}(m)$ and $\widetilde{\beta}^{\rm IR,Upper}(m)$ following $\sqrt{m}$ as shown in **Proposition 2** and **Proposition 3**, respectively. We also observe that $\widetilde{\beta}^{\rm IR}(m) \geq \widetilde{\beta}^{\rm CC}(m) \geq \widetilde{\beta}^{\rm Re}(m)$. While the incremental redundancy scheme outperforms the others, the Chase combining scheme shows better performance than simple retransmission since the desired signals in different retransmission slots are coherently combined. These relations can be employed to provide QoS guarantees in reliable local broadcasting of distributed protocols without the aid of acknowledgement feedback (e.g. ACK/NACK), such as in neighbor discovery, local information exchange in distributed network optimization, and gossip-based algorithms. For example, under Chase combining scheme with the chosen system parameters ($\lambda = 10^{-3}$, $R = 4$, $P = 0.1$, $\alpha = 4$, $N = 1$, $\mu_1 = 0.5 \times 10^{-4}$, and $P_1 = 0.2$), we should broadcast a message 5 times to ensure with 80% that more than 4 nodes will decode the message successfully.

In Fig. 5, we study the impact of an SU's transmit power P_{SU} on its effective mean number of neighbors $\widetilde{\beta}_{eff}$ in local broadcasting. The system parameters are set as $\lambda_{SU} = 10^{-3}$, $R_{SU} = 4$, $\alpha = 4$, $\lambda_{PT} = \lambda_{PR} = 10^{-4}$, and $P_{PT} = 0.3$. The optimal transmit power is $P^*_{SU} = 0.19$ when $\tau_{th} = 1.5 \times 10^{-8}$ ($P^*_{SU} = 0.13$ when $\tau_{th} = 1 \times 10^{-8}$). Obviously, when the interference temperature τ_{th} increases, $\widetilde{\beta}_{eff}$ increases since it is less likely to violate the interference temperature constraints at PRs. In Fig. 6, we study the impact of an SU's information rate R_{SU} on its total delivered data rate. The transmit power of an SU is set as $P_{SU} = 0.1$. The optimal information rate is $R^*_{SU} = 2.4$. The system design of the CRN considering the efficiency of energy and data transmission can be adapted to the interference temperature constraint at the primary system.

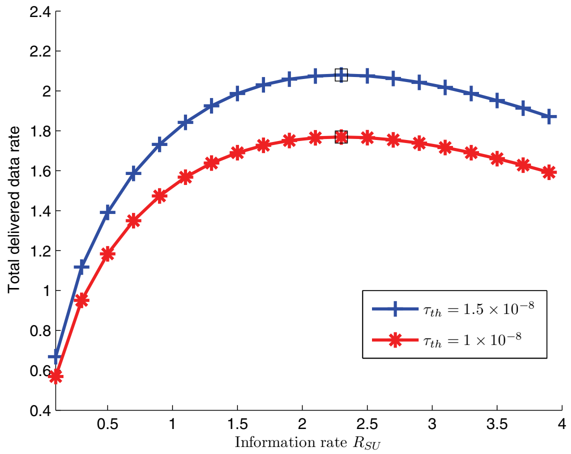

Fig. 6. Plots of the total delivered data rate v.s. information rate of an SU R_{SU}. The system parameters are set as $\lambda_{SU} = 10^{-3}$, $P_{SU} = 0.1$, $\alpha = 4$, $\lambda_{PT} = \lambda_{PR} = 10^{-4}$, $P_{PT} = 0.3$, and $\tau_{th} = 1.5 \times 10^{-8}$ or 1×10^{-8}. All links are Rayleigh faded with unit average power.

VII. Conclusion

In this paper, we study the performance of local single-hop broadcasting in heterogeneous wireless ad hoc networks. Under different error control techniques, the relationships between the mean number of neighbors and the number of retransmissions are established and employed to provide QoS guarantees in reliable local broadcasting. In simple retransmission, the relationship is shown to be $\ln m$, and in both Chase combining and incremental redundancy, the relationships are shown to be m^δ, asymptotically. Finally, the results are applied to local broadcasting in CRN with users of different priorities, where interference temperature is used to restrict the interference from SUs to PUs. The analysis is verified through numerical results, and is useful in facilitating future heterogeneous wireless network design.

Appendix A
Proof of Lemma 2

Let $z \triangleq \frac{gPr^{-\alpha}}{2^R-1}$ (g is a realization of G and r is a constant). For WN_i, $1 \le i \le N$, we partition Ψ_i into two sets $\Xi_i^{(1)}$ and $\Xi_i^{(-1)}$. $\Xi_i^{(1)} \triangleq \{y \in \Psi_i : G_{y0,i}P_i\|y\|^{-\alpha} > z\}$ consists of dominating interferers from WN_i, where a dominating interferer is a node that can cause outage at a typical receiver node of WN_0 with its sole interference contribution. $\Xi_i^{(-1)} \triangleq \{y \in \Psi_i : G_{y0,i}P_i\|y\|^{-\alpha} \le z\}$ is the complement of $\Xi_i^{(1)}$ consisting of non-dominating interferers.

By the mapping theorem [5], the number of nodes in $\Xi_i^{(1)}$ is Poisson distributed with mean

$$\begin{aligned}
&\int_0^\infty \mu_i \mathbb{P}_{G_i}\left(G_iP_i(r')^{-\alpha} > z\right) 2\pi r' dr' \\
&= \mathbb{E}_{G_i}\left[\int_0^\infty \mu_i \mathbf{1}\left(r' < (G_iP_iz^{-1})^{\frac{1}{\alpha}}\right) 2\pi r' dr'\right] \\
&= \mathbb{E}_{G_i}\left[\mu_i \pi (G_iP_iz^{-1})^{\delta}\right] \\
&= \pi\mu_i P_i^\delta \mathbb{E}[G_i^\delta] z^{-\delta} \\
&= \pi\mu_i P_i^\delta \mathbb{E}[G_i^\delta] P^{-\delta} r^2 (2^R-1)^\delta g^{-\delta}. \qquad (25)
\end{aligned}$$

We further define the set $\Xi^{(1)} = \bigcup_{i=1}^N \Xi_i^{(1)}$. $\Xi^{(1)}$ consists of dominating interferers from all $WN_i, 1 \le i \le N$. The number of nodes in $\Xi^{(1)}$ is Poisson distributed with mean

$$\nu_1 = \left(\sum_{i=1}^N \mu_i P_i^\delta \mathbb{E}[G_i^\delta]\right) \pi r^2 (2^R-1)^\delta P^{-\delta} g^{-\delta}. \qquad (26)$$

In addition, the interference from nodes in $\Xi_i^{(-1)}$ to a typical receiver node of WN_0 is defined as $I_i^{(-1)} = \sum_{y\in\Xi_i^{(-1)}} G_{y0,i}P_i\|y\|^{-\alpha}$, and its mean can be computed as

$$\begin{aligned}
&\mathbb{E}[I_i^{(-1)}] \\
&= \mathbb{E}_{G_{y0,i},\Psi_i}\left[\sum_{y\in\Psi_i} G_{y0,i}P_i\|y\|^{-\alpha}\mathbf{1}\left(G_{y0,i}P_i\|y\|^{-\alpha} \le z\right)\right] \\
&\overset{(a)}{=} \int_0^\infty \mu_i \mathbb{E}_{G_i}\left[G_iP_ir^{-\alpha}\mathbf{1}\left(r \ge (G_iP_iz^{-1})^{\frac{1}{\alpha}}\right)\right] 2\pi r dr \\
&= \mathbb{E}_{G_i}\left[\int_{(G_iP_iz^{-1})^{\frac{1}{\alpha}}}^\infty \mu_i G_iP_ir^{-\alpha} 2\pi r dr\right] \\
&\overset{(b)}{=} \frac{2}{\alpha-2}\pi\mu_i P_i^\delta \mathbb{E}[G_i^\delta] z^{1-\delta}, \qquad (27)
\end{aligned}$$

where (a) follows Campbell's theorem [18] and (b) holds for $\alpha > 2$.

We have the following bound

$$\begin{aligned}
&\mathbb{P}\left(\sum_{i=1}^N I_i \le z\right) \\
&= 1 - \left[\mathbb{P}\left(\sum_{i=1}^N I_i > z|\Xi^{(1)} \ne \phi\right)\mathbb{P}(\Xi^{(1)} \ne \phi)\right. \\
&\quad \left. + \mathbb{P}\left(\sum_{i=1}^N I_i > z|\Xi^{(1)} = \phi\right)\mathbb{P}(\Xi^{(1)} = \phi)\right] \\
&= 1 - \left[1\cdot(1-\mathbb{P}(\Xi^{(1)} = \phi))\right. \\
&\quad \left. + \mathbb{P}\left(\sum_{i=1}^N I_i^{(-1)} > z\right)\mathbb{P}(\Xi^{(1)} = \phi)\right] \\
&= \mathbb{P}(\Xi^{(1)} = \phi)\left[1 - \mathbb{P}\left(\sum_{i=1}^N I_i^{(-1)} > z\right)\right] \\
&\overset{(a)}{\ge} \exp(-\nu_1)\left(1 - z^{-1}\sum_{i=1}^N \mathbb{E}[I_i^{(-1)}]\right) \\
&\overset{(b)}{=} \exp(-\nu_1)\left[1 - \left(\sum_{i=1}^N \mu_i P_i^\delta \mathbb{E}[G_i^\delta]\right)\frac{2}{\alpha-2}\pi z^{-\delta}\right] \\
&= \exp(-\nu_1)\left(1 - \nu_1\frac{2}{\alpha-2}\right), \qquad (28)
\end{aligned}$$

where (a) follows Markov's inequality [12] $\mathbb{P}\left(\sum_{i=1}^N I_i^{(-1)} > z\right) \le z^{-1}\mathbb{E}[\sum_{i=1}^N I_i^{(-1)}]$ and ν_1 is defined in (26); (b) follows by (27).

As a result, $\widetilde{\beta}$ can be bounded as

$$\begin{aligned}\widetilde{\beta} &\overset{(a)}{=} \int_0^\infty \lambda \mathbb{P}_{G,I_i}\left(G \geq \frac{(2^R-1)\sum_{i=1}^N I_i}{Pr^{-\alpha}}\right) 2\pi r dr \\ &= \int_0^\infty \lambda \mathbb{E}_G \left[\mathbb{P}_{I_i}\left(\sum_{i=1}^N I_i \leq \frac{GPr^{-\alpha}}{2^R-1} \mid G=g\right)\right] 2\pi r dr \\ &= \mathbb{E}_G\left[\int_0^\infty \lambda \mathbb{P}\left(\sum_{i=1}^N I_i \leq z\right) 2\pi r dr\right] \\ &\overset{(b)}{\geq} \mathbb{E}_G\left[\int_0^\infty \lambda \exp(-\nu_1)\left(1-\nu_1\frac{2}{\alpha-2}\right) 2\pi r dr\right] \\ &\overset{(c)}{=} \frac{\lambda P^\delta \mathbb{E}[G^\delta](1-\frac{2}{\alpha-2})^+}{\left(\sum_{i=1}^N \mu_i P_i^\delta \mathbb{E}[G_i^\delta]\right)(2^R-1)^\delta},\end{aligned} \tag{29}$$

where (a) follows the fourth line in (5), (b) follows by (28), and (c) follows by substituting ν_1 with (26) and evaluating the integral. Note that the lower bound is non-trivial when $\alpha > 4$.

Appendix B
Proof of Proposition 1

Under simple retransmission scheme, we have (30), where (a) assumes that at each retransmission slot k, $1 \leq k \leq m$, the locations of interferers $\Psi_i^{(k)}$ are independently drawn according to a PPP with density μ_i, i.e., the interferers have infinite mobility. (b) follows that with the independence in $\Psi_i^{(k)}$, the expectation commutes with product. (c) follows that the fading gains at retransmission slot k, $G_x^{(k)}$ and $G_{yx,i}^{(k)}$, are exponentially distributed with unit mean.

Appendix C
Proof of Proposition 2

Under Chase combining scheme, we have (31), where (a) is obtained by observing that the ccdf of $\|\mathbf{h}_x\|^2$ is $\overline{F}_{\|\mathbf{h}_x\|^2}(s) = e^{-s}\sum_{k=0}^{m-1}\frac{s^k}{k!}$, whose inverse Laplace transform is $g(t) = \sum_{k=0}^{m-1}\frac{1}{k!}\Delta^{(k)}(t-1)$. With some calculations, we have $\int_0^\infty \frac{g(t)}{t^\delta}dt = \sum_{k=0}^{m-1}\frac{\Gamma(\delta+k)}{\Gamma(\delta)\Gamma(k+1)}$ and $\mathbb{E}\left[\left|\frac{\mathbf{h}_x^H\mathbf{h}_{yx,i}}{\|\mathbf{h}_x\|}\right|^{2\delta}\right] = \Gamma(1+\delta)$. The result follows (5) with the above substitutions.

Appendix D
Proof of Proposition 3

Under incremental redundancy scheme, we have (32), where in (a), we assume, for tractability, that the fading gains of the interfering links are fixed during retransmissions (i.e., $G_{yx,i}^{(k)} = G_{yx,i}$, $\forall k$). (b) follows that $G_x^{(k)}$ and $G_{yx,i}$ are exponentially distributed with unit mean.

Let $S_x^{(k)} \triangleq G_x^{(k)}P\|x\|^{-\alpha}$ and $I_{i,x}^{(k)} \triangleq \sum_{y\in\Psi_i} G_{yx,i}^{(k)}P_i\|y-x\|^{-\alpha}$. Step (a) could be somehow justified when the correlations among $I_{i,x}^{(k)}$ and $I_{i,x}^{(l)}$, $k \neq l$, are high. It can be observed that the temporal correlation coefficient [23], [24] of the signals $S_x^{(k)}$ and $S_x^{(l)}$, $k \neq l$, is zero (uncorrelated). However, under Rayleigh fading channels with static interferers, the temporal correlation coefficient of the interferences $I_{i,x}^{(k)}$ and $I_{i,x}^{(l)}$, $k \neq l$, is 0.5 [23, Corollary 2], indicating that they are somehow correlated and therefore substantiating the assumption.

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$$\begin{aligned}\widehat{\beta}^{\text{Re}}(m) &\overset{(a)}{=} \mathbb{E}_{\Phi,G_x^{(k)},\Psi_i^{(k)},G_{yx,i}^{(k)}}\left[\sum_{x\in\Phi}\left\{1-\prod_{k=1}^{m}\left[1-\mathbf{1}\left(\log\left(1+\frac{G_x^{(k)}P\|x\|^{-\alpha}}{\sum_{i=1}^{N}\sum_{y\in\Psi_i^{(k)}}G_{yx,i}^{(k)}P_i\|y-x\|^{-\alpha}}\right)\geq R\right)\right]\right\}\right]\\
&\overset{(b)}{=} \mathbb{E}_{\Phi}\left[\sum_{x\in\Phi}\left\{1-\prod_{k=1}^{m}\left[1-\mathbb{P}_{G_x^{(k)},\Psi_i^{(k)},G_{yx,i}^{(k)}}\left(\log\left(1+\frac{G_x^{(k)}P\|x\|^{-\alpha}}{\sum_{i=1}^{N}\sum_{y\in\Psi_i^{(k)}}G_{yx,i}^{(k)}P_i\|y-x\|^{-\alpha}}\right)\geq R\right)\right]\right\}\right]\\
&\overset{(c)}{=} \mathbb{E}_{\Phi}\left[\sum_{x\in\Phi}\left\{1-\left[1-\exp\left(-\left(\sum_{i=1}^{N}\mu_iP_i^{\delta}\Gamma(1+\delta)\right)\pi\|x\|^2(2^R-1)^{\delta}P^{-\delta}\Gamma(1-\delta)\right)\right]^m\right\}\right]\\
&= \int_0^{\infty}\lambda\sum_{k=1}^{m}\binom{m}{k}(-1)^{k+1}\exp\left(-k\left(\sum_{i=1}^{N}\mu_iP_i^{\delta}\Gamma(1+\delta)\right)\pi r^2(2^R-1)^{\delta}P^{-\delta}\Gamma(1-\delta)\right)2\pi r dr\\
&= \sum_{k=1}^{m}\binom{m}{k}(-1)^{k+1}\frac{\lambda P^{\delta}}{k\left(\sum_{i=1}^{N}\mu_iP_i^{\delta}\Gamma(1+\delta)\right)(2^R-1)^{\delta}\Gamma(1-\delta)}\\
&= \frac{\lambda P^{\delta}}{\left(\sum_{i=1}^{N}\mu_iP_i^{\delta}\Gamma(1+\delta)\right)(2^R-1)^{\delta}\Gamma(1-\delta)}\sum_{k=1}^{m}\frac{1}{k}\end{aligned} \quad (30)$$

$$\begin{aligned}\widetilde{\beta}^{\text{CC}}(m) &= \mathbb{E}_{\Phi,h_x^{(k)},\Psi_i,h_{yx,i}^{(k)}}\left[\sum_{x\in\Phi}\mathbf{1}\left(\log\left(1+\frac{\|\mathbf{h}_x\|^2P\|x\|^{-\alpha}}{\sum_{i=1}^{N}\sum_{y\in\Psi_i}\left|\frac{\mathbf{h}_x^H\mathbf{h}_{yx,i}}{\|\mathbf{h}_x\|}\right|^2P_i\|y-x\|^{-\alpha}}\right)\geq R\right)\right]\\
&= \mathbb{E}_{\Phi}\left[\sum_{x\in\Phi}\mathbb{P}_{h_x^{(k)},\Psi_i,h_{yx,i}^{(k)}}\left(\log\left(1+\frac{\|\mathbf{h}_x\|^2P\|x\|^{-\alpha}}{\sum_{i=1}^{N}\sum_{y\in\Psi_i}\left|\frac{\mathbf{h}_x^H\mathbf{h}_{yx,i}}{\|\mathbf{h}_x\|}\right|^2P_i\|y-x\|^{-\alpha}}\right)\geq R\right)\right]\\
&\overset{(a)}{=} \frac{\lambda P^{\delta}}{\left(\sum_{i=1}^{N}\mu_iP_i^{\delta}\Gamma(1+\delta)\right)(2^R-1)^{\delta}\Gamma(1-\delta)}\sum_{k=0}^{m-1}\frac{\Gamma(\delta+k)}{\Gamma(\delta)\Gamma(k+1)}\end{aligned} \quad (31)$$

$$\begin{aligned}\widehat{\beta}^{\text{IR, Upper}}(m) &\overset{(a)}{=} \mathbb{E}_{\Phi,G_x^{(k)},\Psi_i,G_{yx,i}}\left[\sum_{x\in\Phi}\mathbf{1}\left(m\log\left(1+\frac{1}{m}\sum_{k=1}^{m}\frac{G_x^{(k)}P\|x\|^{-\alpha}}{\sum_{i=1}^{N}\sum_{y\in\Psi_i}G_{yx,i}P_i\|y-x\|^{-\alpha}}\right)\geq R\right)\right]\\
&= \mathbb{E}_{\Phi}\left[\sum_{x\in\Phi}\mathbb{P}_{G_x^{(k)},\Psi_i,G_{yx,i}}\left(m\log\left(1+\frac{1}{m}\frac{\left(\sum_{k=1}^{m}G_x^{(k)}\right)P\|x\|^{-\alpha}}{\sum_{i=1}^{N}\sum_{y\in\Psi_i}G_{yx,i}P_i\|y-x\|^{-\alpha}}\right)\geq R\right)\right]\\
&\overset{(b)}{=} \frac{\lambda P^{\delta}}{\left(\sum_{i=1}^{N}\mu_iP_i^{\delta}\Gamma(1+\delta)\right)m^{\delta}(2^{R/m}-1)^{\delta}\Gamma(1-\delta)}\sum_{k=0}^{m-1}\frac{\Gamma(\delta+k)}{\Gamma(\delta)\Gamma(k+1)}\end{aligned} \quad (32)$$

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